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Stanley Keith Runcorn

Frank D. Drake

Frank D. Drake, Chief,
Lunar and Planetary Sciences

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The differences in the moments of inertia of the Moon and its ellipticity toward the Earth are between fifteen and forty times those calculated on a hydrostatic theory. Of possible explanations, the most satisfactory explanation is that convection, described by second degree harmonics, is occurring. Second order convection implies the existence of a small core, presumably iron, and results in a new discussion of the Moon's evolution and thermal history.

Author

I. INTRODUCTION

The physics of the interiors of the Moon and the inner planets is studied by the application of scientific inference to observed quantities: mass, radius, moments of inertia, etc. Scientific inference is not a strictly logical process like mathematical deduction but is a synthesis of observations made with the help of previous research experience. The appropriate experience to apply to the task of understanding planetary interiors would be given by the laws of solid-state physics at high temperatures and pressures over long time periods. Thus, the interiors of planets are imperfectly understood partly because of the difficulty of performing laboratory experiments in this field. Consequently, ideas concerning the physical processes within the Earth, inferred from the wealth of geophysical observations, should contribute much to our understanding.

Exploration of the interior of the Moon must begin with Cassini's three laws, which express to a high degree of accuracy observed lunar motion. These laws are as follows:

1. The Moon is a synchronous satellite, which rotates at a uniform rate about an axis fixed with respect to the Moon; the periods of its rotation and orbital motion thus being exactly equal.

2. The inclination of its axis of rotation to the pole of the ecliptic is invariable. This angle is 1°32′.

3. The axis of rotation of the Moon, the pole of the ecliptic, and the pole of the plane of the Moon's orbit always lie in one plane, in the order given. Figure 1 exhibits these laws of the Moon's motion.

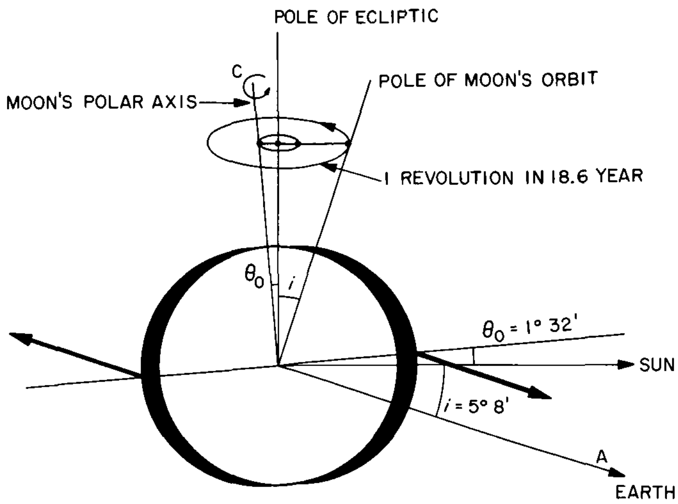

Fig. 1. Cassini's three laws of motion of the Moon

The chief inference to be drawn from these laws is that the Moon possesses a "bulge" toward the Earth, resulting in a difference between the moment of inertia C about its polar axis and the moment A about the mean axis toward the Earth. The Sun's gravitational action being neglected, the first law could be explained by the presence of such a "bulge" and an internal or external source of damping sufficient to reduce the "pendulum" motion to its present imperceptible amount. Alternatively, the first law could be explained if tides raised in the Moon by the gradient of the Earth's gravitational field were displaced from the Earth-Moon axis by a small phase angle through failure of the material to follow the laws of classical elasticity. The torque exerted by the Earth on these tidal bulges would, in due course, change the Moon's rotation to the synchronous condition.

The Sun's chief effect on the Earth-Moon system is to exert a torque, because of the inclination of the Moon's orbit to the ecliptic, causing precession of the nodes of the orbit (the line of intersection of the orbital plane and the ecliptic) in a period of 18.6 years. Although the Earth's equatorial bulge, because its plane is inclined to the Moon's orbit, also results in a similar torque, it is negligible in effect compared with the Sun's action.

II. DYNAMICAL DETERMINATION OF (C–A)/C

The most exact determination of the bulge toward the Earth is obtained by a dynamical argument based on Cassini's laws. $(C–A)/B$ is found to depend on the angle of inclination between the axis of rotation and the pole of the ecliptic. Let θ_0 and i be the inclinations of the lunar equator and orbit to the ecliptic, respectively. Let T be the period of the precession of the node of the Moon's orbit and t the period of lunar rotation. Cassini's second and third laws state that the lunar axis precesses about the pole of the ecliptic in a cone of semiangle θ_0 in a time T. Consequently, at any time the Moon's axis is moving with angular velocity $2\pi\theta_0/T$ in a plane perpendicular to that referred to in Cassini's third law, implying the existence of a torque equal to $C(2\pi/t)\ (2\pi\theta_0/T)$ about an axis perpendicular to the plane defined in Cassini's third law. This torque clearly arises in the main from the attraction of the Earth on the Moon's bulges toward and away from the Earth.

The gravitational potential V of the Moon at a radial distance r from its center of gravity at selenographic coordinates, colatitude θ and longitude ϕ, is given by MacCullagh's formula $V = GM_☾/r + G(A + B + C - 3I)/2r^3 + \ldots$ where I is the moment of inertia about the radius vector. It can be shown (Ref. 1) that

$$V = \frac{GM}{r} + G\left[\frac{(C-A)\ (3\sin^2\theta\cos^2\phi - 1) + (C-B)\ (3\sin^2\theta\sin^2\phi - 1)}{2r^3}\right]$$

We find the gravitational force F acting upon the Earth, when it lies in the $\phi = 0$ plane at colatitude $\theta = 90° \pm (i + \theta_0)$, when the Moon is 90° from the nodes of its orbit. The component of F that is perpendicular to the Earth-Moon axis is given by

$$F = \left(\frac{M}{r}\frac{dV}{d\theta}\right)\ r = R_☾$$

$$= \frac{6GM(C-A)\sin(i+\theta_0)\cos(i+\theta_0)}{2R_☾^4}$$

where M = the Earth's mass. An equal and opposite force acts on the Moon, which experiences a gyroscopic torque. This vanishes when the Moon is at the nodes of its orbit and varies as the square of the sine of its longitude. Consequently, the mean torque on the Moon equals approximately

$$\frac{3GM(C-A)\ (i+\theta_0)}{2R_☾^3}$$

Thus,

$$\frac{(C-A)}{C} = \frac{8\pi^2\theta_0 R_☾^3}{3tTGM(i+\theta_0)}$$

Now, Kepler's third law gives

$$\frac{R_☾^3}{t^2} = \frac{G(M_☾ + M)}{4\pi^2}$$

and if μ is the ratio (t/T) of the lunar period to the period of precession of the node

$$\frac{(C-A)}{C} = \frac{2\theta_0\mu}{3(i+\theta_0)}\cdots \tag{1}$$

Tisserand (Ref. 2) gives the more exact formula from lengthy analysis

$$\theta_0 = \frac{3i\beta}{(1+\mu)\ (2\mu + \mu^2 - 3\beta)\ (1 + \mu + \frac{1}{2}\mu^2)^{1/2}}$$

where $\beta = (C-A)/B$, but if μ and μ^2 are neglected with respect to 1, the same result as Eq. (1) is obtained, to about two significant figures. $(C–A)/B$ is determined to be 0.0006279 (Ref. 3). This is often called the dynamical ellipticity; for if the Moon is taken to be an ellipsoid of revolution (with its long axis toward the Earth), the ratio $(C–A)/B$ is equal to the fractional difference between the axes, by Routh's rule. However, it is also found that A and B are not equal, and the difference may also be found dynamically by the method described in the next section.

III. DETERMINATION OF (C–B)/A

A consequence of Cassini's second law is that the north and south polar regions of the Moon are seen, preferentially, half a lunation apart, as shown in Fig. 1. This is known as the geometrical libration in latitude. A further geometrical libration in longitude occurs in which the Moon rotates relative to its orbital path and, consequently, regions of the limb are alternatively presented to the observer, an effect arising from the ellipticity of the Moon's orbit. By Kepler's second law, the angular velocity of the radius vector between the Earth and Moon, varies inversely as the square of the Earth-Moon distance. This means that after traversing that part of the orbit farthest from the Earth, the Moon will have rotated about its axis more than the angle required to keep the same face toward the Earth. Thus, more of the east limb will be observed, and vice versa, half a lunation later.

The geometrical libration in longitude thus causes the bulge to oscillate monthly with respect to the line of the Earth and Moon, and, consequently, there will be an additional oscillation due to the attraction of the Earth on the Moon's earthward bulge. This is known as the physical libration and has an amplitude of 1 deg. Jeffreys (Ref. 3) reviewed the observations, which are difficult to obtain but which yield a value of $\alpha = (C-B)/A$. Jeffreys found $\alpha/\beta = 0.64$. Earlier observations have given values ranging between 0.49 and 0.78 (Ref. 4).

IV. OPTICAL DETERMINATION OF THE MOON'S NONHYDROSTATIC FIGURE

Baldwin (Ref. 5) considered the apparent displacement of small sharp craters from the center of the lunar disc resulting from the geometrical librations. He separated the points into those of the maria and those of the uplands and fitted to each set of points a parabolic relation, depending on the distance from the center of the disc. In this way he determined that the mean surface was an ellipsoid, with an ellipticity of 0.00130 ± 0.00012 for the uplands and 0.00117 ± 0.00021 for the maria, which corresponds to a bulge of 2.2 km toward the Earth. On the basis of 218 more points, Baldwin (Ref. 6) revised the value to 1.9 km.

The reality of this optically determined bulge has been doubted, mainly because of the considerable scatter in the plotted results. It may be simply calculated that the angular size of the Airy disc for small telescopes used in such studies is about 10^{-6} radian, corresponding to a distance 0.38 km on the Moon's surface. A more important source of scatter will arise from the irregular topography of the lunar surface and also from the variations in illumination of the craters. However, none of these causes of scatter would appear to be systematic over the surface as a whole; consequently, in spite of the considerable scatter, the large number of measurements give a small probable error corresponding to about 0.2 km. However, the most convincing argument for the reality of the bulge arises from the fact that the two values for it, determined from the maria and uplands data, agree within the probable error. Further, Baldwin's method shows that the mean value of the height of the uplands is 1.74 km greater than that of the maria, showing that the method is capable of detecting the existence of an expected mean difference in elevation of these two distinctly different surfaces.

Thus, the bulge toward the Earth is twice that calculated from the difference in the moments of inertia, supposing in the latter calculations the Moon to be an ellipsoid of uniform density. We may thus infer that the latter assumption is wrong and that the Moon's interior is systematically less dense beneath the bulge. It is thus seen to be physically meaningless to calculate a dynamical ellipticity (Ref. 1).

V. THEORIES OF THE NONHYDROSTATIC FIGURE OF THE MOON

The hydrostatic theory of the figure of the Moon supposes it to act as a fluid under the effects of its monthly rotation and the gradient of the Earth's gravitational field. Jeffreys (Ref. 7) shows that, on this theory, the radius toward the Earth would be about 57 m greater than the average radius of the limb and that the equatorial radius in the plane of the sky would be 16 m less than the polar radius. Thus, if the Moon were in hydrostatic equilibrium, it would be, to the accuracy we are concerned with, a perfect sphere. In attempting to explain the observed bulge, Laplace (Ref. 8) and Jeffreys (Ref. 9) were both influenced by the hypothesis, which seemed entirely secure, that the Earth and Moon had once been molten, presumably as the result of a gaseous origin, and were, therefore, naturally led to explanations that the bulge was acquired during solidification. Darwin (Ref. 10) calculated that a uniform ellipsoid with a 1 km bulge would have a maximum stress difference of 20 bars at the center, and, prior to creep studies, it did not seem necessary to question that the Moon could have retained this for billions of years.

Laplace argued that during the cooling of a large body, in which slight inhomogeneities may exist, strains would develop, and he explained the bulge as a natural result of these. Jeffreys (Ref. 9) considered the possibility that it was acquired as the Moon finally solidified in an orbit very much closer to the Earth than the present one, where the tidal action of the Earth would be much greater. The bulge could have been acquired only if the Moon were a synchronous satellite at that time. Thus, Kepler's third law gives the angular velocity of the Moon's rotation as inversely proportional to the three-halves power of the Earth-Moon distance. Thus, the bulge due to the Moon's rotation is proportional to the cube of the Earth-Moon distance: but the tidal bulge has the same dependence. Consequently, Jeffreys' explanation depends on the observed equatorial bulge and the earthward bulge having the same ratio as that on the hydrostatic model, that is 1:4. The observed ratio has not yet been established, as determination of the possible ellipticity of the limb is difficult, partly due to the topography. However, the ratio $(C-B)/(C-A)$, known as the mechanical ellipticity, is by Routh's rule the required ratio. It is 0.6 to 0.7, as shown above.

Urey, Elsasser, Rochester (Ref. 11) propose a different theory. They suggest that, on the accretion theory, iron and stony objects plowing into the Moon's surface could produce differences in density. They require, of course, systematically less dense material in the direction toward the Earth, and it is clear that this could not be brought about by random accumulations, as postulated by the accretion theory.

VI. EXPLANATION OF THE NONHYDROSTATIC SHAPE OF THE MOON BY CONVECTION

Solid-state theory leads one to the conclusion (Ref. 12) that self diffusion processes are bound to cause flow in solids at the reasonably elevated temperatures, which must exist in the Moon if it has even a fraction of the radioactivity found in chondritic meteorites. If the bulge of the Moon had gradually subsided over 4.6 billion years, the flow would have occurred at a rate of strain of 10^{-20} sec^{-1}. This should be compared with the slowest rates of creep (10^{-8} to 10^{-9} sec^{-1}), which are studied in the laboratory. Over such lengths of time it seems permissible to treat the Moon, except for an outer shell in which the temperature is much lower, as a fluid in which the natural cause of motion will be thermally driven convection. The mathematical theory of thermal convection in sphere and spherical shells has been given by Chandrasekhar (Ref. 13), who applies the method first used by Rayleigh in the study of convection in a layer of fluid under a vertical gravitational field.

The Navier-Stokes equation for steady motion, when the velocity v is small so that the acceleration term may be omitted and when the viscosity is high so that the Coriolis force may be neglected, is

$$\mu \nabla^2 \mathbf{v} = \text{grad } p + \mathbf{g} \alpha \rho T$$

when μ is the viscosity, α the volume coefficient of expansion and p and T the pressure and temperature perturbations, g the gravitational field (nearly proportional to the radius for the Moon, the constant of proportionality being $4\pi G\rho/3$, where ρ is the nearly uniform density of the silicates within the Moon).

It is possible to expand a solenoidal vector, such as $\mathbf{v}$ (because the silicates can be treated as incompressible up to the pressures experienced in the Moon), in terms of toroidal and poloidal vectors.

Thus, $\mathbf{v}$ = curl $\mathbf{A}$ where $\mathbf{A}$ is a vector potential and then $\mathbf{A} = \mathbf{r}U + \mathbf{r}_\wedge \text{grad } W$ where U and W are scalar functions whence $\mathbf{v} = \mathbf{r}_\wedge \text{grad } U + \text{curl}^2\, \mathbf{r}W$. The first term is a toroidal vector, and the second is a poloidal vector. As the toroidal vector is perpendicular to the radial vector, it is suitable only for representing rotational motions. The convection motions are then given by

$$\mathbf{v} = \text{curl}^2\, \mathbf{r}W = \text{grad}\left(\frac{d}{dr}(rW)\right) - \mathbf{r}\nabla^2 W$$

The function W is written as a series $W_l(r)S_l(\theta,\phi)/l(l+1)$ where $S_l(\theta,\phi)$ is a surface harmonic of degree l.

The radial velocity $v_r = W_l(r)S_l(\theta,\phi)/r$. Application of vector calculus (Ref. 14) gives the temperature-perturbation.

$$T = -K\nabla^4 W$$

where

$$K = \frac{3\mu}{4\pi G\alpha\rho^2}$$

or

$$T = -K\, D_l^2\, W_l(r) S_l(\theta,\phi)\big/ l(l+1)$$

where

$$D_l = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}$$

Thus, a density deficiency below the Moon's earthward bulge could arise from a convection pattern described by a second degree harmonic, in which the radial velocity would be upward under the bulge and downward in the limb region. (For the moment, for simplicity, the limb is supposed to have no ellipticity and $B = A$.)

It can be shown (Ref. 14) that for a convection pattern, the W of which is a single harmonic, to satisfy boundary conditions regarding pressure, a distortion of the surface is required to a height proportional to the same harmonic. This can be seen physically, for if the viscosity is made infinite the distortion simply results from the hotter and less dense columns being extended. To a first approximation, this distortion can be replaced as a positive surface distribution of matter $m_0 S_l(\theta,\phi)$ and $m_l S_l(\theta,\phi)$ at the outer boundary $r = 1$ and the inner spherical boundary $r = \eta$ respectively.

It can now be proved exactly that in order to explain the differences in the Moon's moments of inertia the convection pattern *must* be of second degree. The differences in the moments of inertia dC, dB, dA about the axes x, y and z and those for a sphere in hydrostatic equilibrium, can then be calculated from the following:

$$dC = \int_0^\pi \int_0^{2\pi} \int_{r=\eta}^{r=1} r^2 \sin^2\theta\, \rho\alpha T S_l(\theta,\phi) r^2 \sin\theta\, d\theta\, d\phi\, dr$$

$$+ \int_0^\pi \int_0^{2\pi} (m_1\eta^4 + m_0)\sin^2\theta S_l(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$dB = \int_0^\pi \int_0^{2\pi} \int_{r=\eta}^{r=1} r^2(1 - \sin^2\theta\sin^2\phi)\rho\alpha T S_l(\theta,\phi)\; r^2 \sin\theta\, d\theta\phi\, dr$$

$$+ \int_0^\pi \int_0^{2\pi} (m_1\eta^4 + m_0)(1 - \sin^2\theta\sin^2\phi) S_l(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

and

$$dA = \int_0^{\pi}\int_0^{2\pi}\int_{r=\eta}^{r=1} r^2(1-\sin^2\theta\cos^2\phi)\rho\alpha T S_l\ (\theta,\phi) r^2 \sin\theta d\theta d\phi\, dr$$

$$+ \int_0^{\pi}\int_0^{2\pi} (m_1\eta^4 + m_0)(1-\sin^2\theta\cos^2\phi)\, S_l(\theta,\phi)\, \sin\theta d\,\theta\, d\phi$$

It is easy to see that dC, dB, and dA reduce to zero unless $S_l(\theta,\phi)$ is a zonal or sectorial harmonic of second degree.

$$dC = 2\pi\left[m_1\eta^4 + m_0 + \rho\alpha\int_{\eta}^{1} Tr^4\, dr\right]\int_0\int_0\left[\frac{2}{3}(1 - P_2(\theta))\right]S_l(\theta,\phi)\sin\theta\ d\theta d\phi$$

$$dA = 2\pi\left[m_1\eta^4 + m_0 + \rho\alpha\int_{\eta}^{1} Tr^4\, dr\right]\int_0^{0}\int_0^{0}\left[\frac{1}{3}(2 + P_2(\theta)) - \frac{1}{6}P_2^2(\theta)\cos 2\phi\right]\ S_l(\theta,\phi)\ \sin\theta\ d\theta\ d\phi$$

$$dB = 2\pi\left[m_1\eta^4 + m_0 + \rho\alpha\int_0^{1} Tr^4\, dr\right]\int^{\pi}\int^{2\pi}\left[\frac{1}{3}(2 + P_2(\theta)) + \frac{1}{6}P_2^2(\theta)\cos 2\phi\right]\ S_l(\theta,\phi)\sin\theta\ d\theta\ d\phi$$

where P_2 and P_2^2 are, respectively, the Legendre function of second degree and associated Legendre function of the second order and second degree.

This theorem is quite general. Whether convection is present or not, the density distribution must contain a second degree surface harmonic; or, if the Moon is uniform, its shape must have a second degree term. Goudas (Ref. 19) has made a spherical harmonic analysis of the Moon's topography and finds the fourth harmonic to be greater than the second. He argues that the former can explain the differences in moments of inertia. The above analysis shows this to be incorrect. But Goudas does not separate the points in the maria and the uplands. As Baldwin (Ref. 5) shows these to be different in height, Goudas' analysis obscures the question of the reality of the bulge and simply describes the positions of the maria and uplands.

VII. DETERMINATION OF FLOW DIRECTIONS WITHIN THE MOON

The difference between A and B is thus dependent on a sectorial harmonic of second degree. A tesseral harmonic $\ell = 2$, $m = 1$ makes no contribution to any moment of inertia. Let

$$S_l(\theta,\phi) = P_2(\theta) + pP_2^2(\theta)\cos 2\phi$$

Thus,

$$\frac{C-B}{C-A} = \left[\frac{-\frac{4}{15} - \left(\frac{2}{15} + \frac{p8}{5}\right)}{-\frac{4}{15} - \left(\frac{2}{15} - \frac{p8}{5}\right)}\right] = \left(\frac{1-4p}{1+4p}\right)$$

and if we take this ratio to be 0.64,

$$p = 0.055$$

It is now possible to determine that the velocities over any spherical surface within the Moon for the southward velocity v_θ along lines of longitude will be proportional to $dS_l(\theta,\phi)/d\theta$, and the eastward velocity v_ϕ along lines of latitude will be proportional to $dS_l(\theta,\phi)/\sin\theta\, d\phi$.

$$P_2(\theta) = \frac{1}{2}(3\cos^2\theta - 1) \quad \text{and} \quad P_2^2(\theta) = 3\sin^2\theta$$

Thus, on any spherical surface within the Moon

$$\frac{v_\theta}{v_\phi} = \frac{[\sin 2\theta - 2p \sin 2\theta \cos 2\phi]}{4p \sin\theta \sin 2\phi}$$

From this expression the flow pattern is determined and plotted on the orthographic projection of the Moon, as shown in Fig. 2. It is, however, not unique, as a tesseral harmonic P_2^1 could have been included: nevertheless, the main features of the motion—an uprising current in the center of the disc—are determined.

Although the main features of the lunar surface are meteoritic, as shown by Baldwin (Ref. 5 and 6), some small domes and lines of small craters may be volcanic. A supposed grid system is also held to exist, e.g., by Fielder (Ref. 15). Certain linear features are also thought to be transcurrent faults. It will be interesting to see whether the distribution of such indications of internal processes within the Moon bear any relationship to the flow pattern shown in Fig. 2. However, the recognition of these features and their definite separation from externally produced features has only just commenced (Ref. 6).

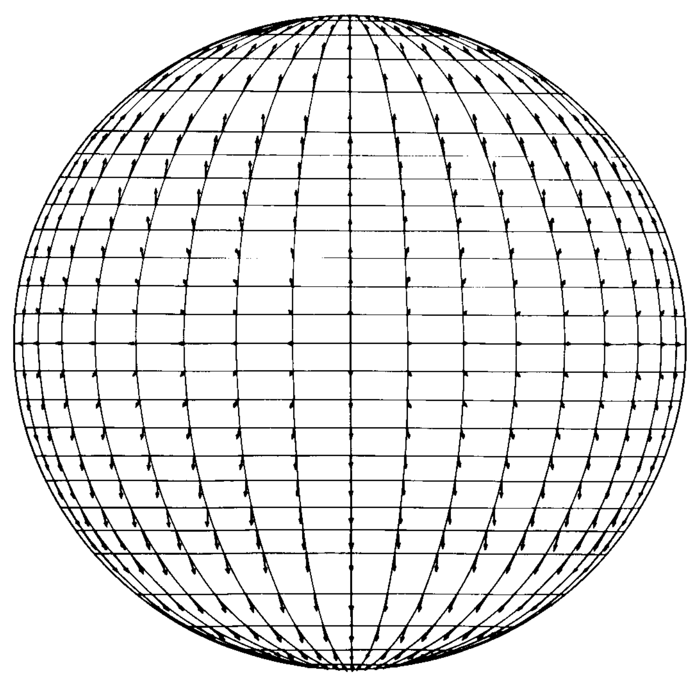

Fig. 2. Zenithal orthographic projection (equatorial case) of the Moon showing distribution of horizontal velocity-components. Lengths of arrows are proportional to flow speeds

VIII. THE EXISTENCE OF A LUNAR CORE

Chandrasekhar (Ref. 13) discussed the convection pattern produced at marginal stability in a fluid enclosed in a spherical shell. He showed that the degree of the harmonic which will establish itself is dependent on the ratio η of the inner and outer radii. In a fluid sphere a first degree harmonic is produced. The second degree harmonic convection pattern, which we have inferred to exist in the Moon, would establish itself at marginal stability only if there is in the Moon a core, the radius of which lies between certain limits given by the mathematical theory. The core surface must be a discontinuity and, therefore, must represent a chemical change. By analogy with the Earth, and bearing in mind the cosmic abundance of the elements and the observed frequent falls of iron meteorites, an iron core is suggested. An alternative explanation is a core formed by a phase change.

The critical values of the ratio of radii η have been computed from Chandrasekhar's curves (Ref. 16). As the core must be small, we may assume that in the Moon the gravitational acceleration g is proportional to the radius. The critical values then depend only on the boundary conditions. The outer crust of the Moon is evidently rigid because of its relatively low temperature. As the creep processes, which allow flow to occur, are likely to depend exponentially on temperature, it seems reasonable to suppose that at a certain depth in the Moon flow becomes possible. In the Earth, various lines of evidence (oceanic heat flow, continental drift, etc.) also imply the existence of a surface, about 100 km deep, below which slow flow occurs and above which the crust has sufficient rigidity to support the topographical features. The crust may fracture and produce fault systems. It will be shown that in the Moon this rigid crust is likely to be thicker.

Chandrasekhar shows that the critical radii depend on whether the outer and inner surfaces are rigid or free. As the melting point of iron is less than that of olivine over the pressure range within the Moon, we take the core surface to be free, i.e., not capable of withstanding shearing stress. The core radius then is found to lie between 0.06 and 0.36 of the radius of the outer surface of the convecting shell. (Had we taken the core to be rigid the values would have been 0.02 and 0.31.)

Such a core has a volume between 0.02 and 4.7% of the Moon's volume. This is satisfactory in that Urey gives the uncompressed mean density of the Moon as 3.4 at ordinary temperatures, and, thus, the mean value of the density of the silicates in the Moon under ordinary temperatures and pressures would be up to 4% less, i.e., 3.3.

A considerable amount is now known about the semiconducting properties of the silicate minerals, particularly in olivine. Semiconduction seems to vary sharply with the amount of ferrous iron (as does the density). Measurements of the fluctuating magnetic fields near the Moon could be used to give valuable information on the radial variation of its internal electrical conductivity as has been done for the Earth.

IX. GRAVITATIONAL ENERGY RELEASED IN CORE FORMATION

It is natural to assume that, if the Moon were formed by accretion, it began 4.6 billion years ago, with the iron uniformly distributed through the silicate. We, therefore, must now consider the gravitational energy released by the separation of the iron to the center of the Moon.

Suppose the outer and inner radii of the Moon's mantle to be R_1 and R_2, respectively. Let the mean density of the Moon be ρ_m, (silicate and iron being ρ_s and ρ_I respectively), and let the variation of these densities with the moderate pressures inside the Moon be neglected. Then the gravitational energy E_0 required to form the Moon initially would be

$$\int_0^{R_1} \frac{4\pi G \rho_m r^2}{3} 4\pi r^2 \rho_m \, dr = \frac{16\pi^2 G \rho_m^2 R_I^5}{15}$$

$$= 3.978 \times 10^{30} \text{ joules}$$

and to form the present core $(16\pi^2/15)\ (G\rho_I{}^2 R_2^5)$. The gravitational field in the present mantle of the Moon is given by

$$\frac{4\pi G}{3} \rho_s r^2 + \frac{4\pi G\ (\rho_I - \rho_s)\ R_2^3}{3r}$$

Consequently, the energy required to form the present mantle of the Moon would be

$$\int_{R_2}^{R_1} \frac{4\pi G}{3} \rho_s r^2 4\pi r^2\ \rho_s dr + \int_{R_2}^{R_1} \frac{4\pi G}{3} \frac{(\rho_I - \rho_s) R_2^3}{r} 4\pi r^2 \rho_s \, dr$$

$$= \frac{16\pi^2 G}{15} \rho_s^2\ (R_1^5 - R_2^5) + \frac{8\pi^2 G}{3} (\rho_I - \rho_s)\, \rho_s R_2^3\ (R_1^2 - R_2^2)$$

Thus, the gravitational energy of the present Moon is

$$E = \frac{16\pi^2 G}{15} \rho_s^2 (R_1^5 - R_2^5) + \frac{8\pi^2 G}{3} (\rho_I - \rho_s) R_2^3 (R_1^2 - R_2^2) + \frac{16\pi^2 G}{15} \rho_I^2 R_2^5$$

where $R_1 = 3.476 \times 10^8$ cm

$$\rho_m = 3.34 \text{ g/cm}^3$$

$$\rho_I = 7.78 \text{ g/cm}^3$$

$$\rho_s = \frac{R_1^3 \rho_m - R_2^3 \rho_I}{R_1^3 - R_2^3}$$

If R_2 equals $\frac{1}{10} R_1$, E is equal to 3.981×10^{30} joules. If R_2 equals $\frac{1}{5} R_1$, E is equal to 4.000×10^{30} joules. If R_2 equals $\frac{3}{10} R_1$, E is equal to 4.055×10^{30} joules. Therefore, the total energy released in the separation of the core from an originally uniform Moon cannot exceed 7.7×10^{28} joules.

X. QUANTITATIVE DISCUSSION OF THERMAL CONVECTION IN THE MOON'S MANTLE

The equation of steady, slow motion in the Moon's mantle is the following:

$$\mu \nabla^2 \mathbf{v} = \text{grad } p + \mathbf{g} \rho \alpha T$$

Where p and T are the pressure and temperature perturbations respectively, $\mathbf{v}$ the velocity, $\mathbf{g}$ the gravity and α the volume coefficient of expansion and ρ the density.

Taking the value of the bulge as 2 km and α as $2.10^{-5}/°C$ we find that the temperature of the upgoing stream is systematically greater than that of the descending stream by about 50°C. The heat transported by convection is of the order of $\rho \sigma T v$ per cm^2 where σ is the specific heat.

Urey (Ref. 17) gives the total present heat generation in chondritic meteorites as 1.6×10^{-7} joule/gm/year and 2330 joules/gm since 4.5×10^9 years ago. Thus, if the Moon (except for the small core) is assumed to have the same radioactivity as the chondritic meteorites, the total heat generated in this way, since its origin, would be 1.7×10^{29} joules (mass of Moon $= 7.36 \times 10^{25}$ gm), just over twice as great as that produced by the separation of the core. Thus, the total heat generated in the Moon since its origin, on these hypotheses, is 2.5×10^{29} joules.

On the accretion hypothesis the initial temperature of the Moon can be regarded as 0°C throughout, as the adiabatic gradient is negligible. Consequently, if no heat escaped, the temperature would have risen to about 2500°C, and, consequently, the Moon would have melted.

However, the action of the convection currents in the Moon are to keep the temperature roughly constant, for the adiabatic gradient in the Moon (which must be exceeded if convection is to occur) can be neglected. The heat $\rho\sigma Tv$ per cm^2 carried by the convection currents must be conducted through the Moon's rigid shell to escape.

We will now argue by analogy with the theory of the Earth's evolution suggested by Runcorn (Ref. 18). It can be assumed that initially the Moon's interior was at too low a temperature for the creep processes to be important: thus, no convection occurred. Neglecting the cooling by conduction, the Moon heated up to 1000°C at 3 billion years ago when convection began. After convection commenced there was a rigid shell of thickness d, below which the Moon was at almost uniform temperature. We must, therefore, assume that a high proportion of the heat generated at present in the Moon by radioactivity, about 10^{11} cal/sec, is conducted out through the surface (the area of which is $3.5 10^{17}$ cm^2). Thus, the thermal gradient at the surface of the Moon would be less than a quarter that of the Earth 0.3 microcal/cm^2/sec. The temperature gradient in the crust of the Moon would, therefore, be about 8°C/km, and, consequently, the depth at which flow becomes possible would be at least three times greater than in the Earth, say 200 km. Here we see a reason why tectonic movements on the Moon's surface are so much less in evidence than on the Earth's. The velocity of the currents in the mantle of the Moon is easily calculated to be of the order of 0.2 cm/yr.

The viscosity of the Moon will be of the order of $g\rho\sigma TL^2/v$, where L is the typical length scale of the motions. This gives a viscosity of the order of 10^{23} poise. It may be noticed that as the temperature is nearly constant in the convecting region and the pressure varies only moderately, a constant value of viscosity is to be expected: we are much less sure of this for the Earth's mantle.

XI. THE NATURE OF THE UPLANDS

It is reasonable to conclude that the Moon's uplands are less dense than the Moon's mantle, for, they must be isostatically compensated if the mantle is capable of flow. By analogy with the Earth, the density of the uplands is taken to be that of granite, 2.7. Then the mean thickness of the uplands is found to be about 9.6 km, using Baldwin's value of 1.74 km as the mean difference in height of the maria and uplands. It may be assumed that convection has been the fundamental mechanism in causing the sialic material to come to the surface. Assuming that 50% of the Moon's surface is covered by "continents," it is simple to calculate that the ratio of surface area to mass of mantle of the Moon and Earth are in the ratio 1 to 3, roughly that of the thickness of the continents on the Moon and Earth. These considerations, therefore, show that assigning a sialic composition to the uplands would not be unreasonable.

XII. CONCLUSIONS CONCERNING THE TEMPERATURE DISTRIBUTION IN THE MOON

Figure 3 shows the inferences drawn in this Report concerning the temperature distribution within the Moon. I represents the region of rapid rise in temperature with depth in which the heat flow is by conduction. III represents the region of convection and IV the core. As in the Earth, there is a region II where the temperature-depth curve relation must be sharply curved. Thus, because in region III the temperature gradient is negligible, region II is where the Moon most closely approaches the melting point of silicates. The question naturally arises whether this is the region from which the lava comes to produce the maria. If so, the maria must have developed much later than has previously been considered.

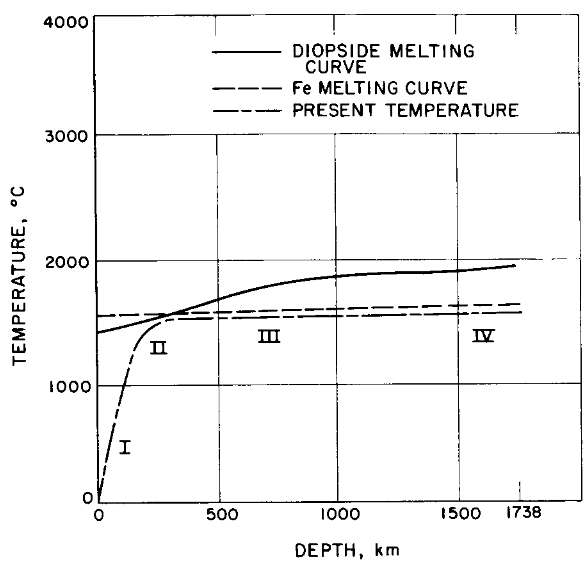

Fig. 3. Temperature distribution within the Moon

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